

# ***Top Quark Mass and Width from Threshold Scan at Linear Collider***

**Alexander Penin**

*University of Alberta & TTP Karlsruhe*

**Snowmass Energy Frontier Workshop**

*Brookhaven National Laboratory, April, 2013*

# Topics discussed

- *Brief introduction*
- *Status of theoretical analysis*
- *Review of experimental simulations*
- *Open questions*

prepared in collaboration with Aurelio Juste (*Universitat Autònoma de Barcelona*)

# Why threshold scan at LC?

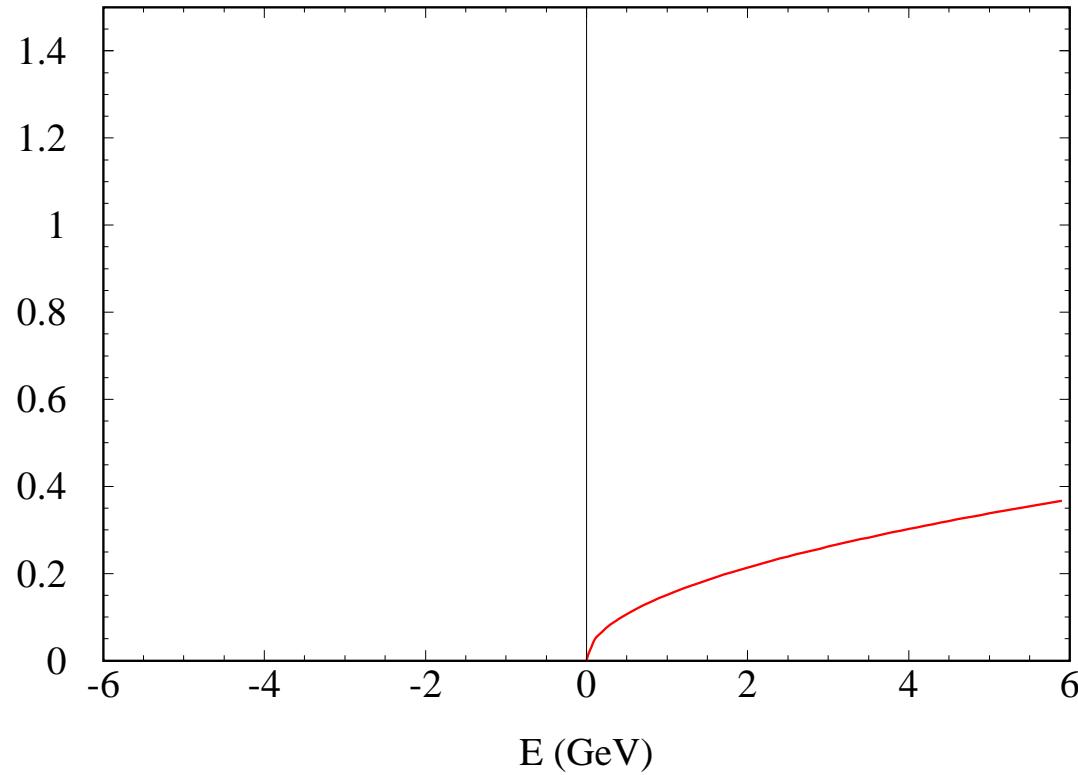
- Precise determination of top mass
    - *crucial for the future of our universe*
  - Traces of new physics in top production and decays
    - *many talks at this meeting*
- “Low energy” high precision experiment

# Why threshold scan at LC?

- Theory
  - *top quark width is a natural infrared cutoff*
  - *first principle QCD predictions*
- Experiment
  - *as clean as possible for a strongly interacting particle*
- Phenomenology
  - *most precise determination of top quark properties such as mass, width, vector couplings*

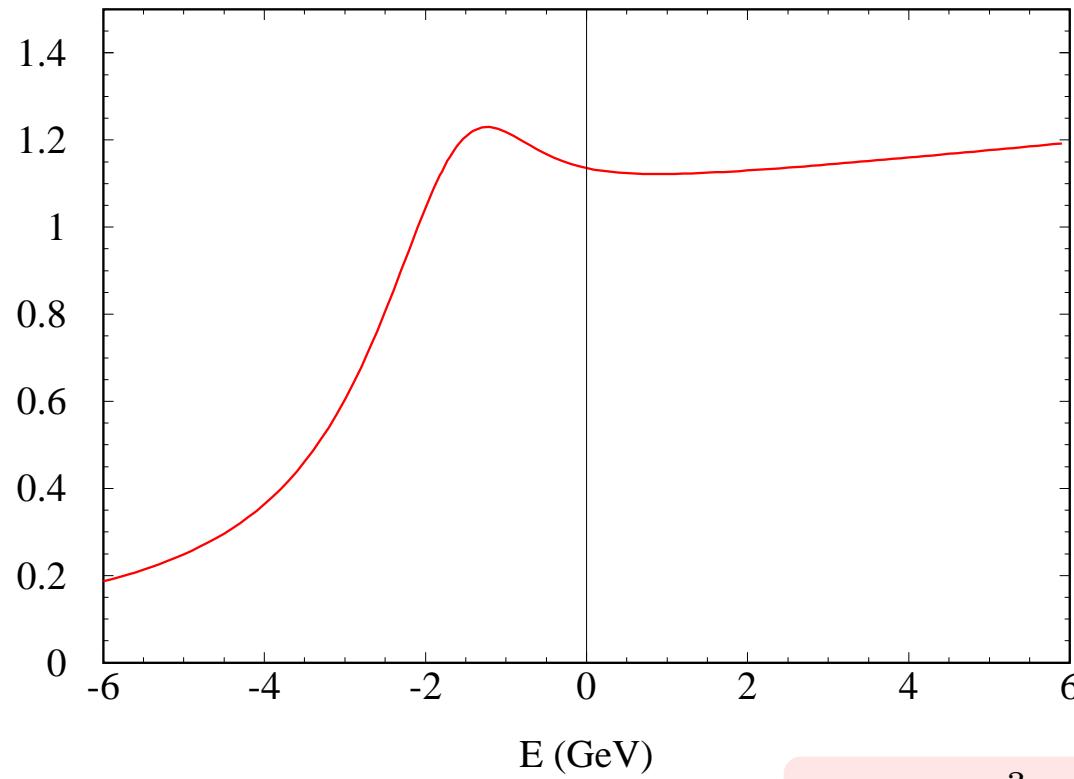
# Born cross section

$\sigma(e^+e^- \rightarrow t\bar{t})$



# Coulomb and finite width effects

$\sigma(e^+e^- \rightarrow t\bar{t})$



E (GeV)

$$\sigma_{\text{res}} \sim \frac{\alpha_s^3}{m_t \Gamma_t}, \quad E_{\text{res}} \sim \alpha_s^2 m_t$$

# Perturbation theory

- NNLO (end of last century)

- *Apparent slow convergence*

- Possible reasons:

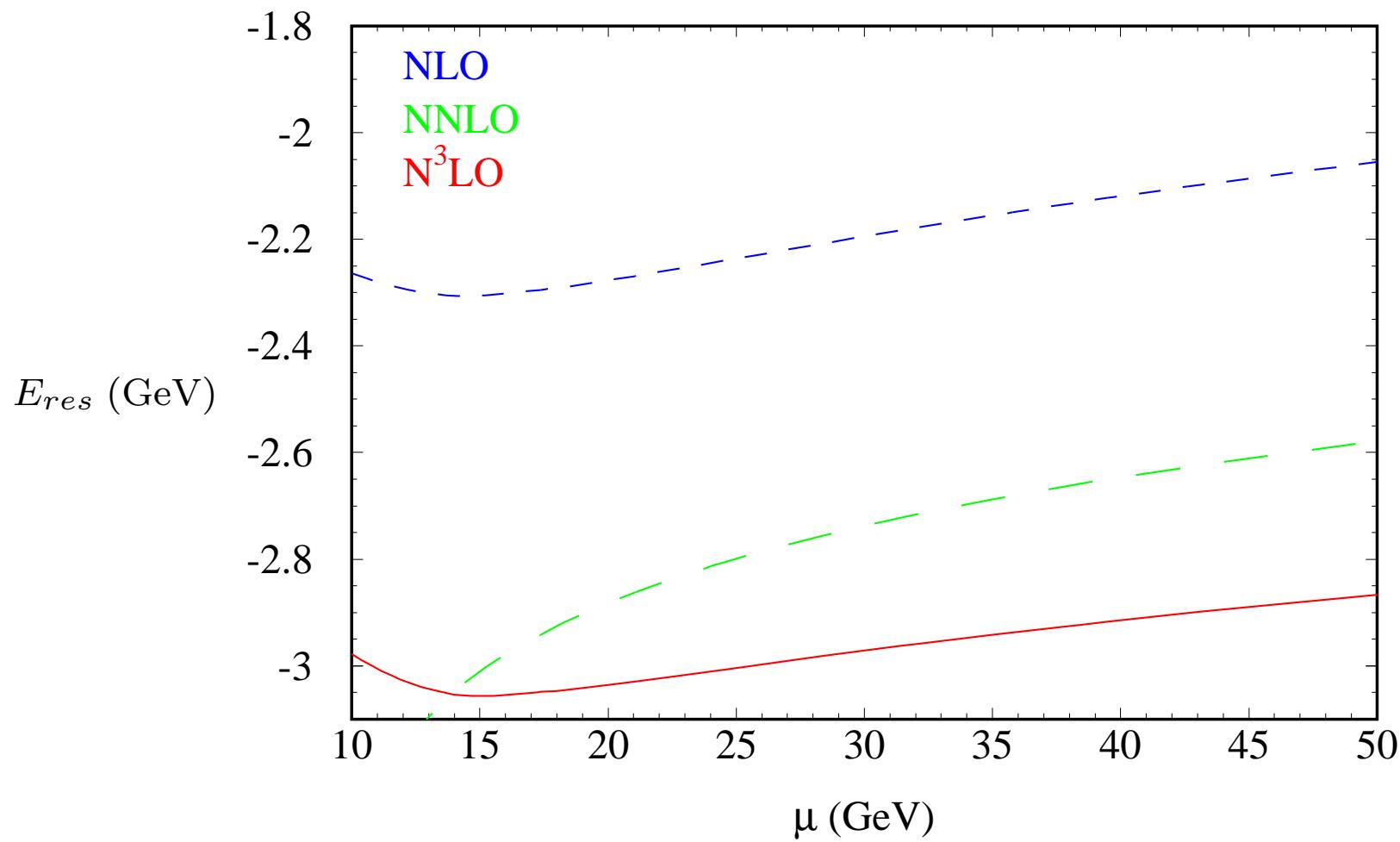
- *Renormalons*  $n!(\beta_0 \alpha_s)^n$

- *Threshold logs*  $\alpha_s^n \ln^m \alpha_s$

→ ***Full  $N^3LO$  analysis is mandatory***

# Top mass at N<sup>3</sup>LO

A. Penin, M. Steinhauser Phys.Lett. B538 (2002) 335



# Top mass at N<sup>3</sup>LO

A. Penin, M. Steinhauser Phys.Lett. B538 (2002) 335

Toponium resonance energy:  $\sqrt{s_{res}} = 2m_t + E_{res}^{N^3LO} + \delta^{\Gamma_t} E_{res}$

$$\sqrt{s_{res}} = \left( 1.9833 + 0.007 \frac{m_t - 174.3 \text{ GeV}}{174.3 \text{ GeV}} \pm 0.0009 \right) \times m_t$$

- *Top quark pole mass  $m_t$  with 80 MeV accuracy*

# Top mass at N<sup>3</sup>LO

A. Penin, M. Steinhauser Phys.Lett. B538 (2002) 335

Toponium resonance energy:  $\sqrt{s_{res}} = 2m_t + E_{res}^{N^3LO} + \delta^{\Gamma_t} E_{res}$

$$\sqrt{s_{res}} = \left( 1.9833 + 0.007 \frac{m_t - 174.3 \text{ GeV}}{174.3 \text{ GeV}} \pm 0.0009 \right) \times m_t$$

- *Top quark pole mass  $m_t$  with 80 MeV accuracy*
- *? renormalons ?*

# Renormalon cancelation

$$\sqrt{s_{res}} = 2m_t - |E_{res}|$$

- Binding energy ( $\mu = C_F \alpha_s m_t$ )

$$E_{res} = E_{res}^{LO} \left[ 1 + 3.201\alpha_s + 16.434\alpha_s^2 + (15.300 \ln \alpha_s + 84.8585) \alpha_s^3 \right]$$

- Mass relation ( $\mu = \bar{m}_t$ )

$$m_t = \bar{m}_t \left[ 1 + 0.424413 \alpha_s - 0.834538 \alpha_s^2 + 2.3682 \alpha_s^3 + 10.5880 \alpha_s^4 \Big|_{\beta_0^3} \right]$$

# Renormalon cancelation

$$\sqrt{s_{res}} = 2m_t - |E_{res}|$$

- Binding energy

$$E_{res} = E_{res}^{LO} \left[ 1 + 3.201\alpha_s + 16.434\alpha_s^2 + (15.300 \ln \alpha_s + 84.8585) \alpha_s^3 \right]$$

- Mass relation

$$m_t = \bar{m}_t \left[ 1 + 0.424413 \alpha_s + 0.834538 \alpha_s^2 + 2.3682 \alpha_s^3 + 10.5880 \alpha_s^4 \Big|_{\beta_0^3} \right]$$

$n! \beta_0^n$  terms

# Renormalon cancelation

$$\sqrt{s_{res}} = 2m_t - |E_{res}|$$

- Binding energy

$$E_{res} = E_{res}^{LO} \left[ 1 + 3.201\alpha_s + 16.434\alpha_s^2 + (15.300 \ln \alpha_s + 84.8585) \alpha_s^3 \right]$$

- Mass relation

$$m_t = \bar{m}_t \left[ 1 + 0.424413 \alpha_s + 0.834538 \alpha_s^2 + 2.3682 \alpha_s^3 + 10.5880 \alpha_s^4 \Big|_{\beta_0^3} \right]$$

$n! \beta_0^n$  terms

# Renormalon cancelation

$$\sqrt{s_{res}} = 2m_t - |E_{res}|$$

- Binding energy

$$E_{res} = E_{res}^{LO} \left[ 1 + 3.201\alpha_s + 16.434\alpha_s^2 + (15.300 \ln \alpha_s + 84.8585) \alpha_s^3 \right]$$

- Mass relation

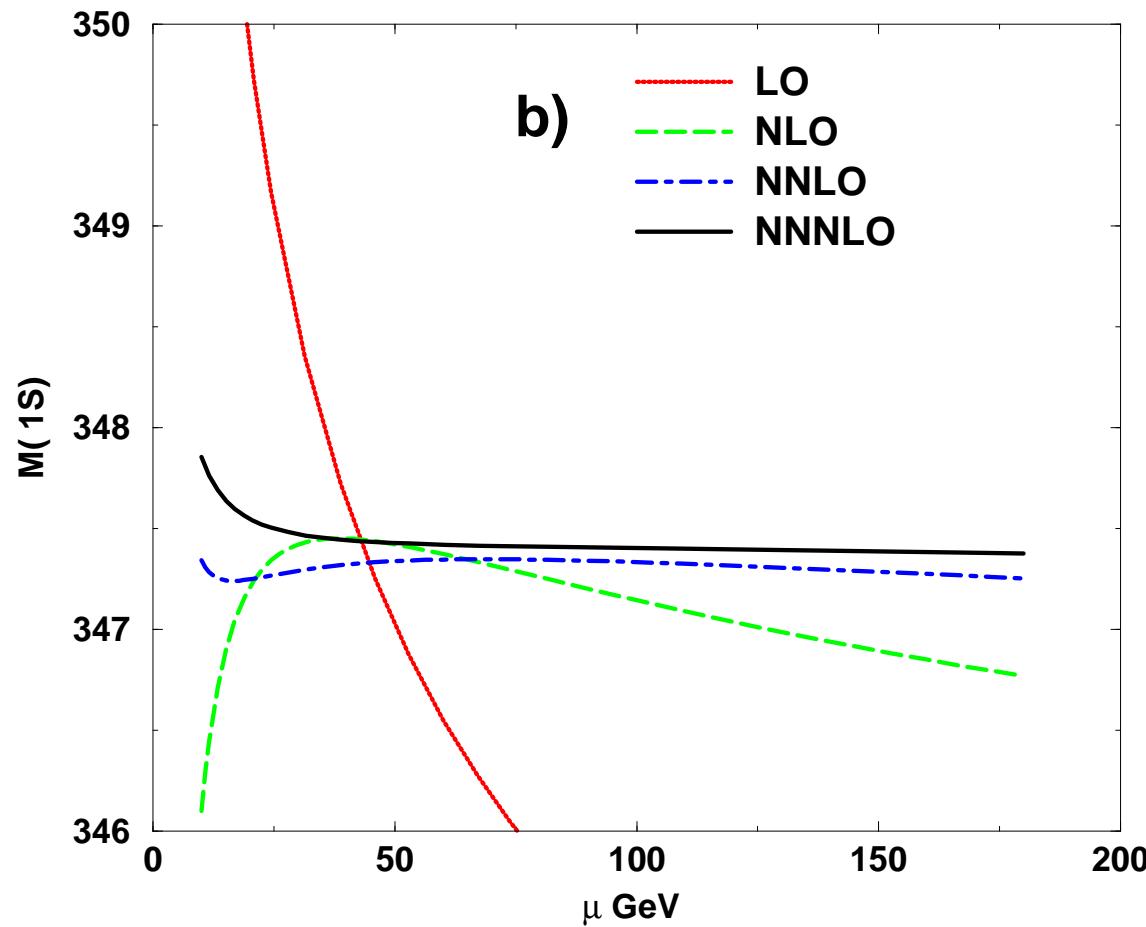
$$m_t = \bar{m}_t \left[ 1 + 0.424413 \alpha_s + 0.834538 \alpha_s^2 + 2.3682 \alpha_s^3 + 10.5880 \alpha_s^4 \Big|_{\beta_0^3} \right]$$

$n! \beta_0^n$  terms

# Top mass at N<sup>3</sup>LO

- Top quark  $\overline{\text{MS}}$  mass  $\bar{m}_t(\bar{m}_t)$  with 50 MeV accuracy

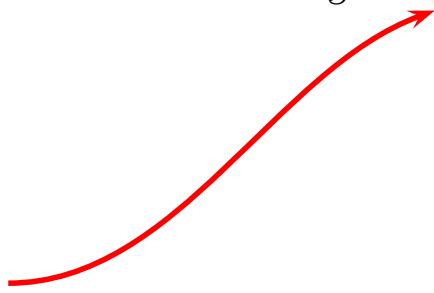
(Y. Kiyo, Y. Sumino, Phys. Rev. D 67 (2003) 071501 )



# Structure of N<sup>3</sup>LO binding energy

$$\delta^{\text{N}^3\text{LO}} E_{res} = \alpha_s^3 \left( 58.205 + 15.297 \ln \alpha_s + 26.654 \right) E_{res}^{\text{LO}}$$

*Renormalon  
contribution*



# Resonance cross section

$$R_{res} = R_{res}^{LO} \left[ 1 - 6.695\alpha_s + (-71.620 \ln \alpha_s + 82.659) \alpha_s^2 \right. \\ \left. + (-16.352 \ln^2 \alpha_s - 0.379 \ln \alpha_s - 7x.xxx) \alpha_s^3 \right]$$

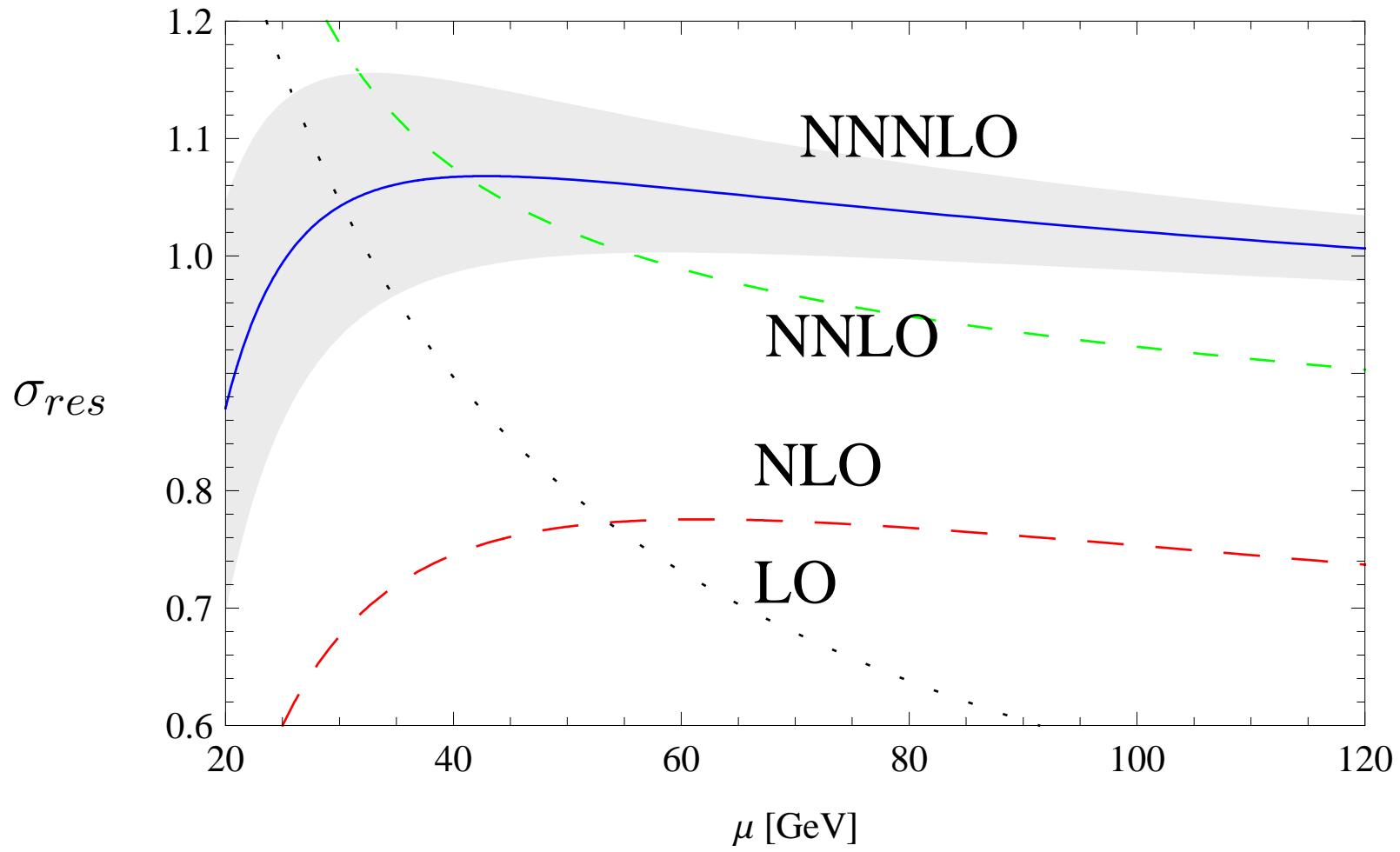
Alberta (A. Penin *et al.*)

Karlsruhe (M. Steinhauser *et al.*)

Munich (M. Beneke *et al.*)

# $N^3LO$ resonance cross section

M. Beneke Y. Kiyo, A. Penin, K. Schuller, arXiv:0710.4236 [hep-ph]



# Finite width effect

- Resonant approximation

V.Fadin, V.Khoze, JETP Lett. 46 (1987) 525

$$\delta(\mathbf{p}^2 - m_t E) \rightarrow \frac{1}{\pi} \frac{\Gamma_t}{(\mathbf{p}^2/m_t - E)^2 + \Gamma_t^2},$$

*not consistent in pNRQCD beyond LO!*

- Nonresonant contribution (*up to 10%*)

- NLO

M. Beneke, B. Jantzen, P. Ruiz-Femenía, Nucl. Phys. B840 (2010) 186

- NNLO

A. Penin, J. Piclum, JHEP 1201 (2012) 034

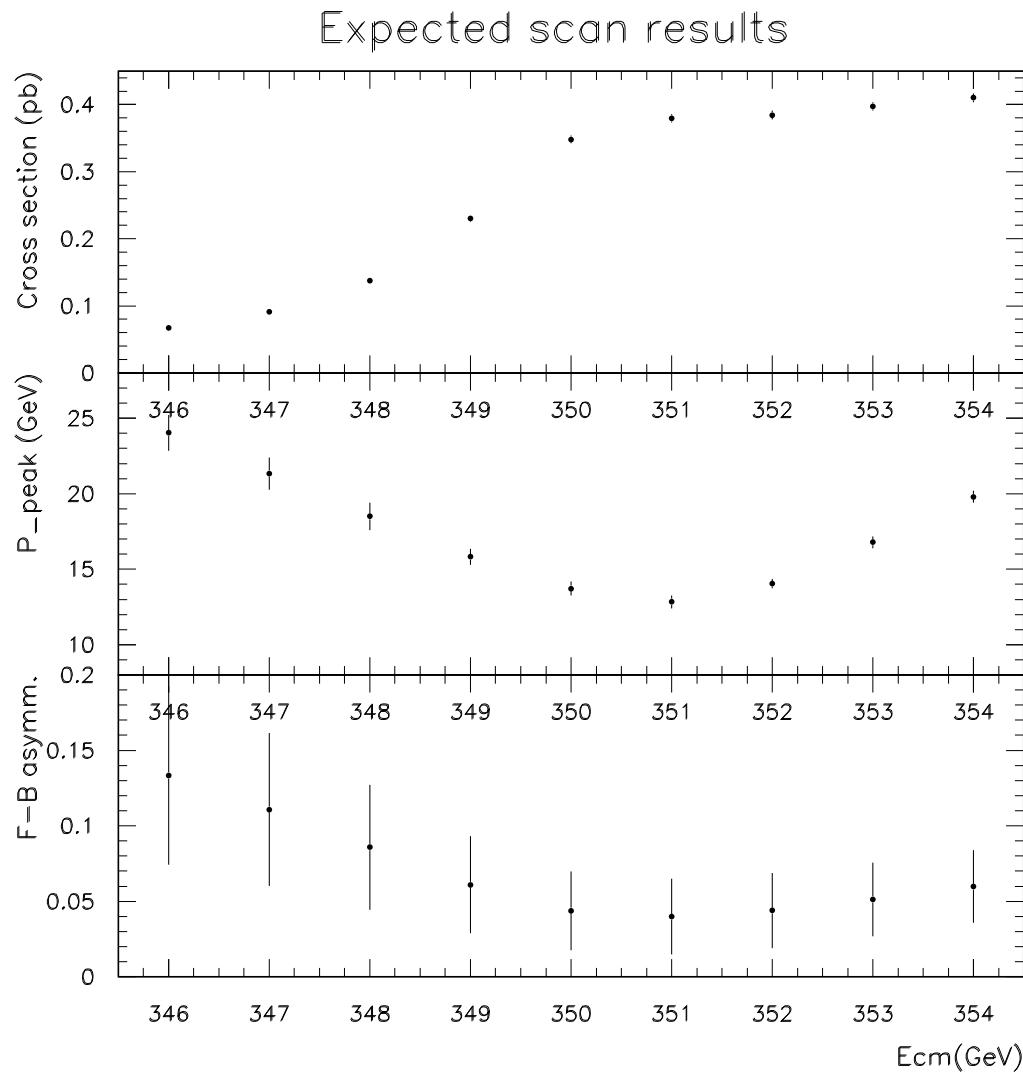
# Theory summary

- Top mass
  - *NNNLO QCD*
  - *80 MeV (50 MeV) accuracy*
- Total cross section
  - *NNNLO QCD (coming soon)*
  - *NNLO finite width*
  - *3% accuracy*
- Differential observables
  - *momentum distribution, forward-backward assymetry*

A. H. Hoang, T. Teubner, Phys. Rev. D **60** (1999) 114027;  
T. Nagano, A. Ota, Y. Sumino, Phys. Rev. D **60** (1999) 114014
  - *NNLO QCD, LO finite width, > 10% uncertainty*

# Simulations

M. Martinez, R. Miquel, Eur. Phys. J. C 27 (2003) 49



# Estimated experimental accuracy

- Detectors, event selection efficiency, statistics ( $300 \text{ fb}^{-1}$ )

M. Martinez, R. Miquel, Eur. Phys. J. C **27** (2003) 49; talk by F. Simon

- *total cross section 3%*
- *differential observables significantly worse*
- *resonance energy 31 MeV*

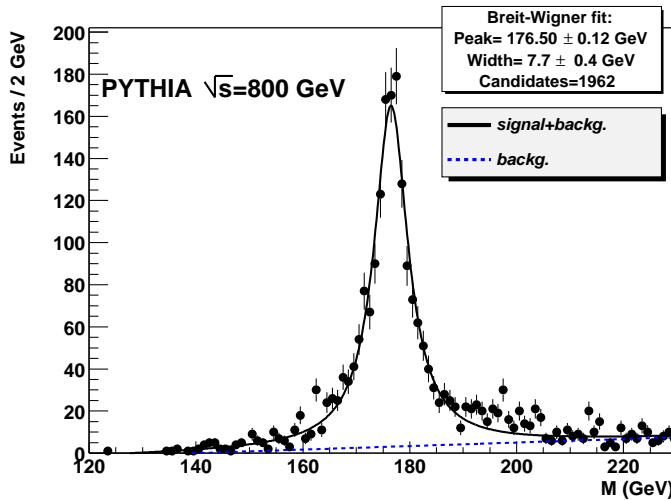
- Beam effects in resonance energy determination

F. Gournaris (PhD thesis 2009)

- *nominal beam energy induced uncertainty 35 MeV*
- *luminosity spectrum induced uncertainty 50 MeV*

# Direct reconstruction of top quark mass

S.V. Chekanov, Eur. Phys. J. C **26** (2002) 13; talk by F. Simon



- Experimental errors in hadronic (semileptonic) channels
  - *statistical*  $100 \text{ fb}^{-1}$ : 100 MeV (140 MeV)
  - *systematical (hadronization model)*: 400 MeV (250 MeV)
- Line shape corrections (A. Denner *et al.*, JHEP **1210** (2012) 110),  $gg \rightarrow t\bar{t}$

# Top precision measurements from threshold scan

✓ Top quark mass

*total uncertainty*  $\sim 100 \text{ MeV}$   $\Rightarrow$  beats direct reconstruction

✓ Top quark width

*total uncertainty*  $\sim 34 \text{ MeV}$

✓ Top quark vector couplings

*total uncertainty*  $\sim 3\%$

✗ Top quark Higgs coupling (from Yukawa potential)

*factor 2 uncertainty*  $\Rightarrow$  cannot compete with Higgs production

# To be done

- Theory

- *Renormalization group improved NNNLO analysis*
- *Four-loop relation between pole and  $\overline{MS}$  mass*
- $\mathcal{O}(\alpha_s)$  *top invariant mass distribution above threshold*

# To be done

- Theory

- *Renormalization group improved NNNLO analysis*
- *Four-loop relation between pole and  $\overline{MS}$  mass*
- $\mathcal{O}(\alpha_s)$  *top invariant mass distribution above threshold*

- Experiment

- *LC operating at 350 GeV*
- *Up-to-date theory input*